A FUNDAMENTAL PROBLEM that hampers progress in improvement of the protection of automobile occupants in collisions is that of measurement of the collision performance of vehicles and protective devices, both experimentally and in the field. Experimental measurements at injury-producing exposure levels require the use of surrogate test subjects and subsequent interpretations of the results in terms of potential injuries to living humans. Field measurements involve real injuries to living humans, but they are complicated by the need to interpret the physical evidence of accidents in order to define the acceleration exposures of the crash victims. The two forms of performance measurement are mutually supportive and both are essential to the achievement of progress.

It is the opinion of the author that the development of techniques for accident investigation has excessively lagged that of techniques for experimental testing. As a result, the effectiveness of even the original protective device, the simple lap belt, has not yet been measured in a rigorous manner (1). Also, the validity of measurements of injury potential with the current, relatively elaborate anthropometric dummies has not been rigorously established (2).

The techniques of investigation of highway accidents have received increasing attention in recent years (3) and (4). However, the achieved investigation results, in terms of the definition of acceleration exposures of crash victims, have continued to reflect wide differences in individual reporting and interpretations of damage and of accident scene information. Analytical reconstruction techniques have been applied in only a limited number of cases. In those cases, the necessity both for selection of appropriate assumptions and approximations and for applications of judgment have tended to produce nonuniform interpretations of the physical evidence. Also, the use of manual calculation procedures has required

Numbers in parenthesis designate References at end of paper.

Correlation of injury with the nature and severity of the acceleration exposure in actual highway accidents is complicated by problems with uniformity in the interpretation of accident evidence. The SMAC and CRASH computer programs have been developed with

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ABSTRACT

the objective of providing aids for interpretation of physical evidence. Through the use of such aids in accident studies, it is possible to establish injury thresholds and mechanisms for living humans in relatively detailed exposures and under different conditions of restraint and protection. In addition to providing refined measures of the performance of protective devices, such studies can provide an improved basis for evaluation of test devices (i.e., anthropomorphic dummies and other surrogate crash victims).

The existing forms and the evidence requirements of the SMAC and CRASH programs are described and results of pilot application studies are presented and discussed. The operating costs of the two programs are defined, and the anticipated future courses of development and application are outlined.
extensive simplifications of analytical relationships, and the accuracy of such manual reconstruction calculations has not been realistically evaluated.

The development of computer aids for accident investigation offers a means of achieving improved uniformity in the interpretation of physical evidence. The Simulation Model of Automobile Collisions (SMAC) and CalSPAN Reconstruction of Accident Speeds on the Highway (CRASH) computer programs are the results of initial efforts directed toward this objective. The degree of success of the SMAC and CRASH development programs in achieving useful and reasonably general investigation aids, from a relatively modest level of developmental effort, indicates that further development, validation and implementation of aids of this type can be highly beneficial toward improving the quality of acceleration-exposure data from accident investigations. In particular, the simpler CRASH program is considered to be a prototype of an inexpensive, universal processor of investigated accident cases. The more complicated, expensive and higher-fidelity SMAC program is seen as a prototype research tool for processing cases of special interest and for guiding further development of the CRASH program.

In the following, the existing forms and the evidence requirements of the two computer programs are briefly described and the results of pilot applications, present limitations, some technical details and future developments are discussed.

SIMULATION MODEL OF AUTOMOBILE COLLISIONS (SMAC) (5)

SMAC is a "simulation" type of program which generates a time-history form of response prediction and a corresponding body of "evidence" (i.e., rest positions, damage and tire marks and tracks) in the same manner as an exploratory physical experiment. Applications follow an iterative procedure in which successive runs are performed, with adjustments of speeds and driver control inputs, until an acceptable overall match of the available real evidence in a given case is obtained. In view of the use of program outputs as the basis for adjusting inputs, SMAC applications constitute a "closed-loop" approximation procedure.

The SMAC computer program consists of a set of equations and associated solution logic defining the behavior of a two-body physical system, which are programmed for simultaneous step-by-step solutions on a digital computer throughout a selected interval of time. The equations define the balance of applied and inertial forces and moments acting on each of the two vehicles during both collision contacts between the vehicles and the separate trajectories that precede and follow a collision. They are based on fundamental physical laws (i.e., Newtonian dynamics of rigid bodies) and on empirical relationships fitted to measured experimental data, such as the load-deflection characteristics of automobile structures and the force-generating properties of tires.

The developed analytical representation of vehicle collisions has been successfully tested by means of direct comparisons of predicted "evidence" with the results of corresponding staged collisions (5) and (6). An optional computer graphics display of predicted results serves to ease the task of making such comparisons (Figure 1).

The operating cost of the SMAC program is approximately $20.00 per individual run. The iterative application procedure generally requires between five and ten individual runs.

CALSPAN RECONSTRUCTION OF ACCIDENT SPEEDS ON THE HIGHWAY (CRASH) (7) and (8)

The CRASH program is a simpler, "closed form" type of calculation procedure (i.e., the program outputs are defined as relatively simple functions of the inputs) which makes direct use of the physical evidence in a given case to produce an approximation of the corresponding impact conditions. It is closely related to the traditional, manual calculations that are performed by accident investigators. The primary difference from manual calculations is in the extent of included details.

For example, manual calculations of the energy dissipated during a "spinout" of a vehicle following a collision (i.e., combined translation and yaw rotation between the separation and rest positions) are generally simplistic and very approximate. The corresponding CRASH calculations are substantially more accurate and, necessarily, are more complex. They include fitted empirical coefficients that have been derived from the results of trial applications to both staged collisions and SMAC runs. In this manner, approximate analytical relationships based on fundamental physical laws for the separate phases of the spinout motion have been adjusted, through the use of fitted empirical coefficients, to yield accurate approximations of the linear and angular velocities at the start of a vehicle spinout. Those velocities, in turn, are applied to conservation of momentum relationships to obtain the velocities at the time of collision contact and the speed changes (ΔV1, ΔV2) experienced by the two vehicles.

The locations and extents of structural deformations on vehicles that have collided provide a separate and independent means of approximating the speed changes experienced
by the two vehicles. Experimental vehicle crush data are stored in the CRASH program for several representative vehicle sizes. Damage dimensions and inertial properties are applied in an approximation of the energy dissipated by vehicle deformation, which is interpreted in terms of the velocity changes of the two involved vehicles.

The CRASH program has been validated by applications to a number of staged collisions (6) and (7). The operating cost is approximately $2.50 per run.

The overall application procedure for CRASH may be viewed as being "open-loop", since the resulting approximation of impact conditions is not tested or adjusted within the existing form of the program. However, an optional output of the CRASH program (full utilization runs) is a complete set of input cards for the SMAC program, with which the CRASH approximation of impact conditions can be tested and refined. A sample computer printout from a CRASH application is presented in Figure 2.

### Fig. 1 - Sample SMAC display

<table>
<thead>
<tr>
<th>RECONSTRUCTED POSITIONS AND VELOCITIES AT IMPACT</th>
<th>DISPLAYED FINAL POSITIONS</th>
<th>VEHICLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.G. POSITION</td>
<td>HEADING</td>
<td>C.G. POSITION</td>
</tr>
<tr>
<td>POS</td>
<td>C.G.</td>
<td>POS</td>
</tr>
<tr>
<td>FT.</td>
<td>FT.</td>
<td>FT.</td>
</tr>
<tr>
<td>DECL.</td>
<td>M/S</td>
<td>DECL.</td>
</tr>
<tr>
<td>VEHICLE 1</td>
<td>10.3</td>
<td>-6.0</td>
</tr>
<tr>
<td>VEHICLE 2</td>
<td>-3.3</td>
<td>3.3</td>
</tr>
</tbody>
</table>

The output of the CRASH program is presented in Figure 2.
programs. It is believed to stem from the fact that CRASH includes an option for interpretation of damage information only, in terms of the corresponding speed changes ($\Delta v_1, \Delta v_2$). However, it should be noted that SMAC can also be applied to the interpretation of incomplete evidence (e.g., damage only), but the costs and complexity of its application generally cannot be justified for cases in which very limited evidence is available. On the other hand, a full utilization of the capabilities of the CRASH program requires the same completeness of evidence as the SMAC program. This fact is obvious when one considers the optional output by CRASH (full utilization runs) of all of the input data required to run the SMAC program.

The relationship of program inputs to accident evidence in the CRASH and SMAC programs should be recognized to clarify the discussion of evidence requirements. In the CRASH program, the inputs are in the form of user responses to questions, and they constitute direct entries of the individual items of accident evidence. In SMAC, the inputs define vehicle properties, tire-terrain friction and a trial set of pre-impact speeds and driver control inputs. The SMAC outputs are compared (by the program user) with the accident evidence. Thus, the manner of using evidence differs in applications of the two programs. However, for full utilization of the capabilities of either program, the evidence requirements are identical.

The minimum amount of physical evidence that can serve as a basis for useful reconstruction calculations in two-vehicle collisions is a definition of the damage to each vehicle and of the sizes of the two vehicles. Note that damage definitions in the form of Vehicle Damage Indices (SAE J224a) may require supplementary dimensional
### Summary of Crash Results

<table>
<thead>
<tr>
<th>Vehicle #1</th>
<th>Vehicle #2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Impact Speed</strong></td>
<td><strong>Impact Speed</strong></td>
</tr>
<tr>
<td>mph</td>
<td>mph</td>
</tr>
<tr>
<td>30.1</td>
<td>46.0</td>
</tr>
<tr>
<td>mph</td>
<td>mph</td>
</tr>
</tbody>
</table>

*   Impact Speed: The speed at which the vehicles collided.
*   Impact Speed: The speed at which the vehicles collided.

<table>
<thead>
<tr>
<th>Vehicle #1</th>
<th>Vehicle #2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Speed Change</strong></td>
<td><strong>Speed Change</strong></td>
</tr>
<tr>
<td>mph</td>
<td>mph</td>
</tr>
<tr>
<td>-0.0</td>
<td>-0.0</td>
</tr>
<tr>
<td>mph</td>
<td>mph</td>
</tr>
</tbody>
</table>

*   Speed Change: The change in speed after the collision.

**DIMENSIONS AND INITIAL CONDITIONS**

**Vehicle #1**

<table>
<thead>
<tr>
<th>A1</th>
<th>80.3 in</th>
<th>A2</th>
<th>80.3 in</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</table>

**Vehicle #2**

<table>
<thead>
<tr>
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<th>80.3 in</th>
<th>A2</th>
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</table>

**WHEEL RESISTANCE**

<table>
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</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>W</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Vehicle #1</th>
<th>Vehicle #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.75</td>
</tr>
</tbody>
</table>

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**Fig. 2B - Output**

Data (e.g., a D category in column 4 does not distinguish between uniform and angled crush profiles). Therefore, measured damage dimensions are considered to constitute a minimum goal in data collection programs where reconstruction calculations are planned. Also, actual or published weights with allowances for the conditions of passenger and/or cargo loading at the time of impact are essential to achieve reasonably accurate results. For the case of minimum evidence, the simplicity of application and the low cost of the CRASH program make it the logical choice.

A more complete definition of the accident evidence for application of either CRASH or SMAC should include as many of the following items as possible.

1. Make, model and year of manufacture of vehicles.
2. Measured weights and/or definition of passenger and cargo loading of each vehicle.
3. Directions of principle forces (estimated from damage).
4. Damage dimensions (see Figure 3).
5. Positions and orientations at rest.
6. Positions and orientations at impact.
7. Rolling resistances of individual wheels of each vehicle (i.e., brake application, engine braking, tire/wheel damage).
8. Tire marks and tracks.
9. Measured friction coefficient(s) and/or description(s) of surface type(s) and condition(s).
10. Definition of boundaries between

**Fig. 3 - Damage dimensions**
Fig. 4 - Injury level vs $\Delta V$ and force direction

terrain zones with different friction properties.

11. Directions and extents of yaw rotations.

12. Evidence of control inputs (i.e., steering, braking, acceleration).

13. Intermediate positions on curved spinout trajectories.

RESULTS OF PILOT APPLICATIONS

As a result of the relatively recent development of the SMAC and CRASH computer programs, the present applications experience is quite limited. In Figure 4, a summary polar plot of injury level as a function of both the magnitude and the direction of the collision speed change, $\Delta V$, is displayed for a small sample of SMAC-reconstructed cases. The data points of Figure 4 are based on reconstructions reported in (9) and (10). The side collisions (9) include both front and rear restrained and unrestrained occupants on the side nearest the collision point in mid-size vehicles. In the frontal collisions (10), the subject occupants were all in the right front seat and only two were restrained ($\Delta V = 28.7 \pm 2^\circ$ and $\Delta V = 12.5 \pm 0^\circ$, each of which is ranked as a minor injury level). The frontal cases were also mid-size vehicles.

The small number of accident cases that is displayed in Figure 4 limits the conclusions that can be drawn regarding definition of injury thresholds. As might be expected, there appear to be substantial overlaps of the exposure levels corresponding to the three displayed injury ratings, with a definite trend toward increased injury as $\Delta V$ increases. A display of this type obviously cannot be expected to be symmetrical or circular, in view of the nature of the passenger compartment configuration and the direct effects of intrusion in some of the side collisions.

A number of more extensive accident studies, using CRASH, are expected to be performed during 1976. With a larger number of data points, it is anticipated that the threshold boundaries for different injury levels will be better defined. Also, when plots can be prepared for given seated positions, conditions of restraint and vehicle sizes, the amount of scatter is expected to diminish. Comparisons of plots for restrained and unrestrained occupants and for different vehicle sizes and types will be of particular interest.

LIMITATIONS OF THE EXISTING FORMS
OF SMAC AND CRASH

In their present forms, both SMAC and CRASH are limited to flat-surface collisions that do not include complicating factors such as secondary collisions with roadside obstacles, effects of terrain features (e.g., gradients, curbs) and rollovers. However, it should be noted that it is presently feasible for a skilled user of the computer programs to handle a number of complicating factors. By means of estimating vehicle velocity at the point of secondary collision, rollover, etc., and then using a corresponding, "equivalent rest position" beyond that point, on the basis of a simple, flat-surface spinout at the estimated velocity, it is possible to extend the generality of the programs. The SMAC program has been applied to multiple collision cases by means of restarting the two-body simulation for each vehicle after separation from one of the collisions. These forms of extension of generality obviously can eventually be automated through the introduction of appropriate simulation features in SMAC and additional user questions in the CRASH program and the coding of corresponding approximation techniques (e.g., the relationships developed for speed estimates in rollovers in (11) could serve this purpose).

An additional limitation of the distributed version of SMAC is that of simulating collisions in which the extent of interference between the two vehicles is very small and the use of simple Coulomb inter-vehicle friction does not yield realistic tangential forces. This shortcoming of SMAC is particularly evident in offset frontal collisions with small overlap where "snagging" of the interacting structures occurs. In the proprietary Ficalvision version of SMAC, a simple simulation of such snagging effects has been successfully incorporated (e.g., see Figure 1, in which that program extension was applied).
INTERPRETATION OF COLLISION DAMAGE

Both the SMAC and the CRASH computer programs include simplified representations of the dynamic crush properties of vehicle structures. In particular, the crush properties are approximated by means of the assumption of elastic-plastic behavior in which the force-deflection characteristic is linear for increasing load. Clearly, this is an aspect of the described computer aids that should be refined as improved experimental data from staged collisions become available. Note that one suggested approach for such a refinement is presented in (12). The benefit/cost relationship of any related program modifications must, of course, be evaluated prior to implementation. However, aside from the question of accurate analytical representation, a more fundamental and recurring question regarding the interpretation of collision damage appears to require some related discussion.

The effect on the extent of damage, for a given speed change, of a non-zero velocity of the striking vehicle subsequent to a collision continues to be a topic about which erroneous and/or misleading statements are frequently made (13, 14 and 15). This topic is addressed in (16). A somewhat expanded discussion and analysis, in which the structural damage produced by a given speed change of a vehicle is shown to be independent of the speed range of the collision, is presented in the Appendix of this paper.

FUTURE DEVELOPMENTS

In the course of future development of the described investigation aids, the distinction between needs of a "production" type program and those of a developmental prototype must be recognized. The introduction of an excessive number of user options and adjustments can work against the original objective of uniformity of results. Therefore, analytical refinements and extensions must be "built-in" and made automatic to the greatest possible extent.

Effects of non-planar terrain features (e.g., gradients, embankments, curbs, etc.) can be incorporated through the use of existing, simplified approximation techniques to estimate the corresponding speed changes during spinout trajectories. By working back from the rest positions through such terrain features, it will be possible to estimate the separation velocities at the end of collision contact.

It is believed that the limited application experience to date has clearly established the feasibility of achieving uniform interpretations of physical evidence in large-scale accident studies through the use of the CRASH program. Obviously the validation of the computer program must be extended as improved data from staged collisions become available. Also, some additional development must be anticipated to extend its generality as additional application experience is gained. The applications that are included in NHTSA research programs during 1976 will serve to identify the complicating factors that are most frequently encountered and, thereby, will provide guidance for needed extensions and revisions.

Development and validation of the SMAC program should be continued so that it can serve as a research tool for accident cases of special interest and can provide a high-fidelity reference for evaluating analytical extensions incorporated in CRASH.

APPENDIX - DAMAGE INTERPRETATIONS

The general nature of recurring erroneous and/or misleading discussions of damage interpretations is indicated in the following paragraph.

For a collision speed change (ΔV) of the striking vehicle of 50 MPH occurring between 100 MPH and 50 MPH, it is asserted that the structure dissipates three times as much energy as in the case of the same speed change occurring between 50 MPH and 0 MPH. Further, it is argued that a substantially greater structural deformation in the former case can lead to an erroneous overestimate of the severity of the occupant exposure. In (14), an impact with a "solidly constructed heavy lorry" is used as an illustrative example of the "nearest real-life equivalent to a moving crash barrier". For a collision of this type, the conclusions of (13) and (14) regarding effects of the speed range in which a given speed change occurs on the extent of energy absorption are clearly erroneous. In (15), statements regarding relationships between energy dissipation and ΔV are made which, for collisions in general, are not true. They are followed by an illustrative example, which is the somewhat special case of a sideswipe against a fixed barrier in which the colliding bodies do not reach a common velocity.

If such discussions and the related conclusions were correct for collisions in general, it would be necessary to know not only the mass and stiffness of the struck object but also the speed range in which the damage occurred before damage to the striking vehicle could be interpreted in terms of occupant exposure severity. Since the SMAC and CRASH computer programs are based on an assumption that the damage for a given ΔV is independent of the speed range in which the ΔV occurs, it was considered to be essential to attempt to clarify the involved physical principles.
Energy Considerations - Perhaps the simplest approach to the topic is a straightforward analysis of the total change in system energy that occurs as the result of a central collision between two bodies. It must be recognized that in order to achieve a 100 MPH to 50 MPH speed change in a non-sideswipe collision, the struck object must be accelerated from its initial speed to the common speed of 50 MPH in the direction of motion of the striking vehicle. Therefore, in Figure 5, the following relationships must exist.

Initial Kinetic Energy of System,

\[(KE)_1 = \frac{1}{2} M_1 V_{10}^2 + \frac{1}{2} M_2 V_{20}^2\]  

(1)

Final Kinetic Energy of System,

\[(KE)_2 = \frac{1}{2} (M_1 + M_2) V_c^2\]  

(2)

From Conservation of Momentum, in a central collision,

\[M_1 V_{10} + M_2 V_{20} = (M_1 + M_2) V_c\]  

(3)

Solution of (3) for the required initial speed of the struck vehicle, \(V_{20}\), as a function of the initial velocity of the striking vehicle, \(V_{10}\), and the common velocity, \(V_c\), yields

\[V_{20} = \frac{1}{M_2} \left[ (M_1 - M_2) V_c - M_1 V_{10} \right].\]  

(4)

Substitution of (4) in (1) and subtraction of (2) from (1) yields the following expression for the total amount of system energy dissipated during the collision.

\[DE = (KE)_1 - (KE)_2 = \frac{M_1}{2} \left( 1 + \frac{M_1}{M_2} \right) \left( V_{10}^2 - V_c^2 \right).\]  

(5)

Since the speed change, \((\Delta V_1)\), experienced by the striking vehicle, \(M_1\), is equal to \((V_{10} - V_c)\), equation (5) may be expressed as,

\[DE = \frac{M_1}{2} \left( 1 + \frac{M_1}{M_2} \right) (\Delta V_1)^2.\]  

(6)

Note that equation (6) is independent of the speed range in which \(\Delta V_1\) occurs.

In the case of an SAE barrier crash,

\[M_2 \to \infty\] and \(V_{20} = 0\).

Therefore, \(V_c = 0\), \((KE)_2 = 0\), and

\[\frac{M_1}{2} \left( 1 + \frac{M_1}{M_2} \right) (\Delta V_1)^2.\]  

(7)

From equations (6) and (7) it may be seen that the total dissipation of system energy in the case of a collision of a given speed change with an obstacle of finite mass (i.e., movable) is related to that for the corresponding case of an SAE barrier crash by the following ratio

\[\frac{DE}{(DE)_B} = 1 + \frac{M_1}{M_2}.\]  

(8)

Note that equation (8) is independent of the speed range in which the energy dissipation (DE) occurs.

The dissipated system energy will be distributed between the two colliding bodies in inverse proportion to their relative stiffnesses. From equation (8), for the case of identical vehicles, a collision in which \(M_1\) decelerates from 100 MPH to 50 MPH will produce an energy dissipation twice as large as that in an SAE barrier crash at 50 MPH. However, since the dissipation will distribute equally between the structures of identical vehicles, each vehicle will be damaged to the same extent as it would be in a 50 MPH barrier crash. Note that the stated speed-change conditions of this sample case would require that the equal-mass struck vehicle be initially at rest. Also, the present discussion is limited to the case of a uniform distribution of structural stiffness in the vertical plane (i.e., the effects of under-ride/override are not included).

A 100 MPH to 50 MPH collision with a heavier vehicle for which \(M_1 \ll M_2\) will produce a smaller total dissipation of system energy than that in the cited equal-mass case (i.e., a larger portion of the initial system energy will become kinetic energy of the struck vehicle) with the limiting value of the total dissipated energy approaching that of a 50 MPH barrier crash (see Figure 6). In this sample case, it would be necessary for the heavier vehicle to be initially moving in the same direction as the striking vehicle (i.e., for a common velocity of 50 MPH to be achieved). Note that the collision force acting on the
heavier vehicle does work by virtue of the fact that the struck vehicle is moving, whereas no work is done on a fixed obstacle.

**Force Considerations** - For a given speed change of the striking vehicle \( \Delta V_1 \) to be produced, the time integral of the net applied force must equal the corresponding momentum change of that vehicle (i.e., Newton's Second Law)

\[
\int_0^t F \, dt = M_1 \Delta V_1 \tag{9}
\]

If it is assumed for simplicity that the force-deflection characteristics of the two vehicles are each approximately linear for increasing load and, further, that tire-terrain forces are negligibly small in relation to the collision forces, the following analysis can provide relationships with which to evaluate the implications of equation (9).

In Figure 7, the symbols are defined as follows:

\( M_1, M_2 = \) Masses of Vehicles 1 and 2, lb sec²/in.

\( K_1, K_2 = \) Peripheral crush stiffnesses of Vehicles 1 and 2 for increasing load, lb/in.

\[ X_1, X_2 = \text{Displacements of centers of mass, inches}. \]

The accelerations of \( M_1 \) and \( M_2 \) can be expressed

\[
M_1 \ddot{X}_1 = -\left( \frac{K_1 K_2}{K_1 + K_2} \right) (X_1 - X_2). \tag{10}
\]

\[
M_2 \ddot{X}_2 = \left( \frac{K_1 K_2}{K_1 + K_2} \right) (X_1 - X_2). \tag{11}
\]

Let \( \delta = x_1 - x_2 \), \( \delta = x_1 - x_2 = v_1 t - v_2 t \), where \( v_{10}, v_{20} = \) initial velocities, inches/sec.

From (10) and (11),

\[
\ddot{\delta} + \frac{K_1 K_2}{K_1 + K_2} \left( \frac{M_1}{M_1 + M_2} \right) \delta = 0. \tag{12}
\]

Solution of (12) yields the following relationships:

**Response Frequency**, \( f = \frac{1}{2\pi} \sqrt{\frac{K_1 (1 + M_1/M_2)}{M_1 (1 + K_1/K_2)}} \) cycles/sec \( \tag{13} \)

From (13), the time to reach a common velocity (i.e., one quarter of a complete cycle) can be expressed as

\[
t = \frac{1}{4} \left( \frac{1}{f} \right) = \frac{\pi}{2} \sqrt{\frac{M_1 (1 + K_1/K_2)}{K_1 (1 + M_1/M_2)}} \text{ seconds} \tag{14}
\]

It may be seen in equation (13) that the response frequency of the two mass system decreases as the obstacle mass (i.e., the struck vehicle mass) increases and/or its stiffness decreases. Therefore, from equation (14), the time to reach a common velocity increases as the obstacle mass increases and/or its stiffness decreases. Note that equation (14) is independent of the speed range in which the collision occurs. The effects of the cited two struck vehicle variables on the time to reach a common velocity are depicted in Figure 8.
\[
\delta_1 = \left( \frac{K_2}{K_1 + K_2} \right) \delta 
\] (16)

From (15) and (16),
\[
(\delta_1)_{\text{max}} = (V_{10} - V_{20}) \sqrt{\frac{K_2 M_1 M_2}{K_1 (K_1 + K_2)(M_1 + M_2)}} \quad \text{inches} 
\] (17)

From Conservation of Momentum, the common velocity, \(V_c\), may be obtained.
\[
V_c = \frac{M_1 V_{10} + M_2 V_{20}}{M_1 + M_2} = \frac{V_{10} + M_1 V_{10}}{1 + M_1/M_2} 
\] (18)

Equation (18) can be solved for the speed change of \(M_1\).
\[
V_{10} - V_c = \frac{(V_{10} - V_{20})}{(1 + M_1/M_2)} 
\] (19)

Substitution of (19) into (17) yields
\[
(\delta_1)_{\text{max}} = (V_{10} - V_c) \sqrt{\frac{M_1 (1 + M_1/M_2)}{K_1 (1 + K_1/K_2)}} \quad \text{inches} 
\] (20)

Since the speed change, \(\Delta V_1\), experienced by the striking vehicle is equal to \((V_{10} - V_c)\), equation (20) may be expressed as
\[
(\delta_1)_{\text{max}} = (\Delta V_1) \sqrt{\frac{M_1 (1 + M_1/M_2)}{K_1 (1 + K_1/K_2)}} \quad \text{inches} 
\] (21)

In the case of a SAE barrier, for which \(M_2 = 0\), \(K_2 = 0\), equation (21) becomes
\[
(\delta_1)_B = (\Delta V_1) \sqrt{\frac{M_1}{K_1}} \quad \text{inches} 
\] (22)

The ratio of deformations for movable/fixed obstacle collisions with a given speed change is obtained from equations (21) and (22).
\[
\frac{(\delta_1)_{\text{max}}}{(\delta_1)_B} = \sqrt{\frac{1 + M_1/M_2}{1 + K_1/K_2}} 
\] (23)

Plots of results obtained with equation (23) are displayed in Figure 9. Note that the plotted relationship is independent of the speed range in which the collision occurs.

The relationships that have been derived and the associated discussion are believed...
to clearly establish the fact that the structural damage produced by a given speed change of a vehicle in a non-sideswipe collision is independent of the speed range in which it occurs. Therefore, interpretations of damage in terms of the severity of occupant exposure can be made without knowledge of the final speed of the striking vehicle. The extent of damage does, of course, depend on the mass and stiffness of the struck obstacle. Also, in a more complete and realistic analysis, the effects of non-central impact configurations must be considered (7, 17).

It appears that the existing confusion related to this topic stems from a lack of recognition of the fact that work is done in accelerating a movable struck obstacle to the common velocity. As a result of this oversight, the total kinetic energy loss of the striking vehicle is erroneously expected to appear in the form of dissipation through structural damage. In view of the use of a collision with a moving truck as an illustrative example in (14), it is speculated that effects of underride may in some cases have been misinterpreted as a confirmation of the expected damage increase. For the special case of a sideswipe, the interpretation of damage must include consideration of the fact that no common velocity was reached.

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REFERENCES


