SAE #970960
Reviewer's Discussion
by James A. Neptune, J_2 Engineering, Inc.
Effects of Restitution in the Application of Crush Coefficients
Raymond R. McHenry, Author

The authors' revised model for restitution takes a step in the right direction. The revised model, however, is not entirely realistic. As a result, it may not correctly represent a vehicle's crush response characteristics.

The revised model for restitution in figure 6A is a completely elastic crush response (ε=1.0) through the first several inches of dynamic crush. Completely elastic dynamic crush is a theoretical condition. In the real world, restitution may approach the theoretical magnitude of 1.0 but will never equal 1.0. Restitution is a mathematical "limit." The limit of the magnitude of restitution is 1.0 as dynamic crush approaches zero. Therefore, it is not realistic for the first several inches of dynamic crush to be completely elastic. In addition, the revised equation for restitution (appendix 1, equation 1) is an "infinite limit," or asymptote, where restitution (ε) becomes infinite as dynamic crush (δ_m) approaches zero. This equation is not realistic. If this model is implemented in a computer program, hopefully the program will provide the necessary limit not present in equation (1).

The crush response of the high restitution hypothetical vehicle in figure 2 is completely elastic through the first 6.2 inches of crush. A completely elastic crush response (ε=1.0) through the first 6.2 inches of crush is not realistic. Clearly front bumpers, with rare exception, do not extend 6.2 inches beyond easily damaged vehicle components such as the fenders and hood (note: any residual damage means $\varepsilon \neq 1.0$). Also "energy absorbing" bumpers that are present on vehicles "absorb energy" and, therefore, are not completely elastic. In addition, the vehicle in figure 2 would begin to develop residual crush (ε<1.0) after the first 5.6 inches of dynamic crush. This means that, between 5.6 inches and 6.2 inches of dynamic crush, the vehicle's crush response is partially plastic (ε<1.0) yet at the same time is completely elastic (ε=1.0). This is impossible.

Considering the above, the conclusions and graphs that are drawn from analyzing the hypothetical vehicles are subject to question (Analytical Approach section, figures 4 & 5). If the revised restitution model can produce a vehicle that could not exist in the real-world, can one be certain that the revised coefficients will correctly represent a vehicle's crush response characteristics when applied to a real-world vehicle?

Figure 2 Calculations:

Use equation (1), set ε=1.0 and solve for δ_m. $$\delta_m = \frac{6.2897}{1.0 + 0.0097} = 6.2 \text{ inches.}$$

Use $K_1 = \frac{4102}{72} = 57$ lb/in² & equation (9), set δ_f =0 and solve for δ_m. $$\delta_m = \frac{317}{56}\sqrt{\frac{56}{57}} = 5.6 \text{ inches.}$$

Or set equation (9)=equation (16), set δ_f =0 and solve for δ_m. $$\delta_m = \frac{6.2897}{\sqrt{\frac{5065}{4102} + 0.0097}} = 5.6 \text{ inches.}$$

The revised coefficients for 1979-82 Ford LTD's (figure 12) indicate that the first 5.7 inches of dynamic crush is completely elastic with no energy being absorbed by the vehicle's structure [use equation (1), set ε=1.0 and solve for δ_m]. Does this correctly represent this vehicle's crush response characteristics?

In figure 13 the revised coefficients for 1981-85 Ford Escort's would indicate the first 2.5 inches of dynamic crush is completely elastic [use equation (1), set ε=1.0 and solve for δ_m] and that a dynamic crush of 7.4 inches would occur before the onset of residual crush [use equation (9), set δ_f =0 and solve for δ_m]. Can this vehicle's front end structure flex rearward through 7.4 inches with no residual damage? Wouldn't the easily damaged vehicle parts such as the fenders and hood sustain significant damaged before 7.4 inches of dynamic crush occurs? Does this correctly represent this vehicle's crush response characteristics? [Note: Equations (1) & (16) can be used to evaluate the crush-restitution relationship in the range of crush that occurs after the onset of residual crush (δ_f >0). Equation (16), however, can not be used for the initial elastic-only portion of dynamic crush (before the onset of residual crush, δ_f) since the ratio δ_f/δ_m is zero. Equation (1), therefore, must be used to evaluate the crush-restitution relationship for the dynamic crush that occurs before the onset of residual crush.]

MacInnis Engineering (SAE Paper 940916) has tested a 1981 Ford Escort. Full stroke of the front bumper isolators is 57 mm (2.25 inches). Residual crush began at a dynamic crush of 55 mm (2.17 inches). Maximum restitution of 0.61 occurred at 0.5 mm of isolator stroking (dynamic crush). The MacInnis data contradicts the revised restitution model. Restitution is not equal to a constant value of 1.0 over the first 2.5 inches of dynamic crush. In addition, the onset of residual crush occurs at a dynamic crush of 2.17 inches rather than 7.4 inches. Analysis of the revised coefficients for the 1975-79 VW Rabbit (figure 14) when compared to data from SAE Paper 940916 produce similar contradictions.

The original CRASH/SMAC programs and the revised CRASH/SMAC programs were used to estimate ΔV for a hypothetical vehicle. The authors characterize the differences (10 to 30%) between the estimated ΔV's of the original versus revised CRASH/SMAC programs as "errors." Differences between the estimated ΔV's would be more accurately characterized as "potential improvements in accuracy" rather than "errors." A characterization of "error" should be limited to comparisons between estimated ΔV's and the actual ΔV's measured during the thorough testing of a real-world vehicle.

In the Introduction section of the paper, the authors state that the original SMAC program inaccurately estimates ΔV approximately 10 to 30% for the case of direct central barrier collisions. There is no reference to the source of this information/conclusion. Hopefully this conclusion is not based upon the above "error" analysis involving hypothetical vehicles. Also in appendix 3, the authors indicate that it is "common knowledge" that the original CRASH program "underestimates the ΔV in barrier crashes by approximately 10% to 20% at 30 MPH and by greater amounts at lower speeds." Stating this is "common knowledge" should not relieve the authors of the responsibility of providing the supporting evidence for this conclusion. The lack of supporting evidence raises questions regarding the accuracy of these numbers.

The authors' revised model for restitution, although a step in the right direction, is not completely realistic. The restitution model needs to be reconfigured such that restitution is less than, or equal to, the theoretical limit of 1.0 at zero dynamic crush ($\varepsilon_0 \leq 1.0$) and decreases from that point as dynamic crush increases.

From SAE publication SP-1237, pages 324-325

This is the Authors' Reply to the erroneous and misguided Reviewer's Discussion for SAE Paper 97-0960 "Effects of Restitution in the Application of Crush Coefficients"

It should have been allowed and included with the misguided and erroneous Reviewer's Discussion. SAE and the session organizers were made fully aware of the situation

Raymond R. McHenry, Brian G. McHenry, Authors

The first paragraph of Mr. Neptune's "discussion" fully states his opinions.

The remainder of his review contains a series of mathematical errors, flawed arguments, and/or deliberate misstatements:

On **page 325 of SP-1237, in paragraph 1**, Mr. Neptune states "Equation (16), however, can not be used for the initial elastic-only portion of dynamic crush (before the onset of residual crush, δ_f) since the ratio δ_f / δ_m is zero.".

Equation (16) from SAE 97-0960 (page 321) is as follows: $$\varepsilon = \sqrt{\frac{E_R}{E_A}} = \left(1 - \frac{\delta_f}{\delta_m}\right)\sqrt{\frac{K_2}{K_1}}$$

It is not clear what problem Mr. Neptune has with Equation (16) when δ_f / δ_m goes to zero. The only calculation 'problem' with equation (16) would occur if $\delta_m = 0.0$ (i.e. where there is no maximum deformation, which occurs only when there is no collision).

Obviously, from elementary mathematics, $$\varepsilon = \sqrt{\frac{K_2}{K_1}} \text{ when } \delta_f = 0.$$

The "impossible" calculation results cited by Mr. Neptune are the direct result of his incomprehensible rejection of equation (16) for the case of $\delta_f = 0$.

Additional responses to Neptune's Reviewer's Discussion contained in SP-1237 are as follows:

p. 324, paragraph 2: Figure 6A depicts the restitution coefficient for a specific defined set of inputs rather than the overall general case. The infinite limit argument deliberately ignores item 3 of Appendix 3.

p. 324, paragraphs 3 through 5: From equation (16), $(\varepsilon)_{max} = \sqrt{\frac{K_2}{K_1}}$. In the specific example in Figure 2,

$\sqrt{\frac{K_2}{K_1}}$ > 1.00. Therefore, $(\varepsilon)_{max}$ is set to 1.000. Setting $(\varepsilon)_{max}$ equal to 1.000 in equation (1) yields ε = 1.000 for $\delta \leq$ 6.2 inches. No mystery. Simple mathematics.
The "not realistic" comments are not supported by any specific identified evidence.

Neptune ignores item 7 of Appendix 3 which points out the fact that CRASH (EDCRASH) effectively assumes an elastic range, in terms of full dimensional recovery, equal to A/B. (For further clarification of this statement, please see Question#4 of "Questions related to 97-0960")

The proposed restitution model has an effective elastic range, in terms of full dimensional recovery, equal to $\frac{A}{\sqrt{BK_1}}$ which is smaller than that of EDCRASH (Equation (9)).

See item 5 of Appendix 3 for a discussion of "absorbed energy", "elastic" terminology.

p 325, paragraph 1: In Figure 13, $(\varepsilon)_{max}$ is established by equation (16) to be 0.314. It makes no sense to set ε equal to a value larger than $(\varepsilon)_{max}$ in equation (1). The relationship between equations (1) and (16) that is defined by Neptune is nonsense.

p 325, paragraph 2: SAE paper 940916, 1981 Ford Escort. A 50 mm tear in the top flange of the front bumper at V= 11.07 MPH. No damage to other front end components. 2.25 inch isolator compression (fully rebounded). Neptune apparently assumes no dynamic deformation other than the isolators.

p 325, paragraph 3: The complete omission of restitution effects in CRASH (EDCRASH) creates an ERROR equal to the magnitude of the ignored restitution effects.

p 325, paragraph 4: In any collection of SAE barrier tests, compare the measured values of Δ V with the approach speeds.

p 325, paragraph 5: Neptune states that "The restitution model needs to be reconfigured such that restitution is less than, or equal to, the theoretical limit of 1.0 at zero dynamic crush and decreases from that point as the dynamic crush increases". The suggestion of a "reconfiguration" deliberately ignores item 4 of Appendix 3, the fact that $(\varepsilon)_{max}$ is presently limited to $\sqrt{\frac{K_2}{K_1}}$ or 1.000, whichever is smaller.